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LETTER TO THE EDITOR

On the Laplace transform of the G function

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Abstract. It is pointed out that the main result of a recent letter on the Laplace transform of Meijer's G function is actually a special case of an already established transform.

In a recent letter, Wille (1986) has stated as the main result his equation (3) which gives the Laplace transform of the G function with argument proportional to x^k, where k is a positive integer. This result however is a special case of the already known result for argument proportional to $x^{k/\rho}$, where k and ρ are positive integers (see § 3.3.6*a* in the listing by Mathai and Saxena (1973) and references therein). The latter is, in turn, a special case of Saxena's Mellin transform of the product of two G functions (see equation (3.2.2) of Mathai and Saxena 1973).

In their book Mathai and Saxena have remarked that 'By specialising the parameters in (3.2.2) and using the tables of the particular cases of the G-function . . . , we obtain many important integrals of the Special Functions . . .'. Instances of the latter are the Laplace transforms of the Airy functions which were calculated by A G Gibbs in response to a problem posed by Smith (1973). These transforms were rediscovered by Davison and Glasser (1982) and provided with alternative proofs by Leach (1983) and Exton (1985) as well as by Wille (1986). The Laplace transforms of the Airy functions $\mathrm{Ai}(-x)$ and $\mathrm{Bi}(-x)$ are tabulated in Apelblat ((1983), § 12.15, equations (19) and (20)); that of $\mathrm{Ai}(x)$ is readily obtained from equation (18) of Apelblat's § 12.15 while for $\mathrm{Bi}(x)$, as is well known, the transform does not exist.

With improving effectiveness of applications in the physical sciences of the special functions and the hypergeometric functions (see Srivastava and Karlsson (1985) and references therein), there is an increasing likelihood of the independent rediscovery of known but difficult to recognise results. To the examples cited above could be added the rediscoveries by Niukkanen (1983, 1984), pointed out by Srivastava and Manocha (1984) and Srivastava (1985). This situation is not only wasteful of effort but illustrates that the full power of the generalised special functions and their known properties are not being exploited. In this context it may be worth reminding readers therefore that, although applications often involve only the simpler members of classes of functions, much may be known of the general case and that, furthermore, the whole class itself is sometimes a special case of even more general functions, also with known properties.

Thus, as discussed above, the Airy functions, etc, are special cases of Meijer's G function; the latter then provides an example of a class which is a special case of a much more general and also widely studied function, namely the H function (Fox's H function and multiple variable versions thereof) (Fox 1961, Gupta and Jain 1966, Mathai and Saxena 1978, Srivastava *et al* 1982, Srivastava and Goyal 1985). While the latter does contain as particular cases most of the special functions used in applied mathematics, it does not contain some of importance; for instance the Riemann zeta function nor indeed the polylogarithm of any order (Marichev 1983, equation (7.33)). Even the H function can be looked at as a special case of the recently proposed (Inayat-Hussain 1986) $\bar{H}$ function which does include the polylogarithm and, as well, the exact thermodynamic free energy of a certain Gaussian model (Joyce 1972) of magnetic phase transitions.

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